

Lepton flavor violating signals of a little Higgs model at the high energy linear e^+e^- colliders

Chong-Xing Yue and Shuang Zhao

Department of Physics, Liaoning Normal University, Dalian 116029, China *

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Abstract

Littlest Higgs (LH) model predicts the existence of the doubly charged scalars $\Phi^{\pm\pm}$, which generally have large flavor changing couplings to leptons. We calculate the contributions of $\Phi^{\pm\pm}$ to the lepton flavor violating (LFV) processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_k$, and compare our numerical results with the current experimental upper limits on these processes. We find that some of these processes can give severe constraints on the coupling constant Y_{ij} and the mass parameter M_Φ . Taking into account the constraints on these free parameters, we further discuss the possible lepton flavor violating signals of $\Phi^{\pm\pm}$ at the high energy linear e^+e^- collider (ILC) experiments. Our numerical results show that the possible signals of $\Phi^{\pm\pm}$ might be detected via the subprocesses $e^\pm e^\pm \rightarrow l^\pm l^\pm$ in the future ILC experiments.

*E-mail: cxyue@lnnu.edu.cn

I. Introduction

It is well known that the individual lepton numbers L_e, L_μ , and L_τ are automatically conserved and the tree level lepton flavor violating (LFV) processes are absent in the standard model (SM). However, the neutrino oscillation experiments have made one believe that neutrinos are massive, oscillate in flavors, which presently provide the only experimental hints of new physics and imply that the separated lepton numbers are not conserved[1]. Thus, the SM requires some modification to account for the pattern of neutrino mixing, in which the LFV processes are allowed. The observation of the LFV signals in present or future high energy experiments would be a clear signature of new physics beyond the SM .

Some of popular specific models beyond the SM generally predict the presence of new particles, such as new gauge bosons and new scalars, which can naturally lead to the tree level LFV coupling. In general, these new particles could enhance branching ratios for some LFV processes and perhaps bring them into the observable threshold of the present and next generations of collider experiments. Furthermore, nonobservability of these LFV processes can lead to strong constraints on the free parameters of new physics. Thus, studying the possible LFV signals of new physics in various high energy collider experiments is very interesting and needed.

Little Higgs models[2] employ an extended set of global and gauge symmetries in order to avoid the one-loop quadratic divergences and thus provide a new method to solve the hierarchy between the TeV scale of possible new physics and the electroweak scale $\nu = 246GeV = (\sqrt{2}G_F)^{-\frac{1}{2}}$. In this kind of models, the Higgs boson is a pseudo-Goldstone boson of a global symmetry which is spontaneously broken at some high scales. Electroweak symmetry breaking ($EW\!SB$) is induced by radiative corrections leading to a Coleman-Weinberg type of potential. Quadratic divergence cancellation of radiative corrections to the Higgs boson mass are due to contributions from new particles with the same spin as the SM particles. This type of models can be regarded as one of the important candidates of the new physics beyond the SM .

The littlest Higgs model (LH)[3] is one of the simplest and phenomenologically viable models, which realizes the little Higgs idea. Recently, using of the fact that the LH model contains a complex triplet Higgs boson Φ , Refs.[4,5,6] have discussed the possibility to introduce lepton number violating interactions and generation of neutrino mass in the little Higgs scenario. Ref.[5] has shown that most satisfactory way of incorporating neutrino masses is to include a lepton number violating interaction between the triplet scalars and lepton doublets. The tree level neutrino masses are mainly generated by the vacuum expectation value (VEV) ν' of the complex triplet Φ , which does not affect the cancellation of quadratic divergences in the Higgs mass. The neutrino masses can be given by the term $Y_{ij}\nu'$, in which Y_{ij} (i, j are generation indices) is the Yukawa coupling constant. As long as the triplet VEV ν' is restricted to be extremely small, the value of Y_{ij} is of natural order one, i.e. $Y_{ij} \approx 1$, which might produce large contributions to some of LFV processes[6,7].

The aim of this paper is to study the contributions of the LFV couplings predicted by the LH model to the LFV processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_k$ and compare our numerical results with the present experimental bounds on these LFV processes, and see whether the constraints on the free parameter Y_{ij} can be obtained. We further calculate the contributions of the LH model to the LFV processes $e^\pm e^\pm \rightarrow l_i^\pm l_j^\pm$ and $e^+ e^- \rightarrow l_i^\pm l_j^\pm$ (l_i or $l_j \neq e$), and discuss the possibility of detecting the LFV signals of the LH model via these processes in the future high energy linear $e^+ e^-$ collider (ILC) experiments.

This paper is organized as follows. Section II contains a short summary of the relevant LFV couplings of the scalars (doubly charged scalar $\Phi^{\pm\pm}$, charged scalars Φ^\pm , and neutral scalar Φ^0) to lepton doublets. The contributions of these LFV couplings to the LFV processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_k$ are calculated in section III. Using the current experimental upper limits on these LFV processes, we try to give the constraints on the coupling constant Y_{ij} in this section. Section IV is devoted to the computation of the production cross sections of the LFV processes $e^\pm e^\pm \rightarrow l_i^\pm l_j^\pm$ and $e^+ e^- \rightarrow l_i^\pm l_j^\pm$ induced by the doubly charged scalars $\Phi^{\pm\pm}$. Some phenomenological analyses are also included in this section. Our conclusions are given in section V.

II. The LFV couplings of the triplet scalars

The LH model[3] consists of a nonlinear σ model with a global $SU(5)$ symmetry and a locally gauged symmetry $[SU(2) \times U(1)]^2$. The global $SU(5)$ symmetry is broken down to its subgroup $SO(5)$ at a scale $f \sim TeV$, which results in 14 Goldstone bosons ($GB's$). Four of these $GB's$ are eaten by the gauge bosons (W_H^\pm, Z_H, B_H), resulting from the breaking of $[SU(2) \times U(1)]^2$, giving them masses. The Higgs boson remains as a light pseudo Goldstone boson and other $GB's$ give masses to the SM gauge bosons and form a scalar triplet Φ . The complex triplet Φ offers a chance to introduce lepton number violating interactions in the theory.

In the context of the LH model, the lepton number violating interaction which is invariant under the full gauge group, can be written as[5,7]:

$$\mathcal{L} = -\frac{1}{2}Y_{ij}(L_i^T)_\alpha \Sigma_{\alpha\beta}^* C^{-1}(L_j^T)_\beta + h.c. \quad (1)$$

Where i and j are generation indices, α and β ($= 1, 2$) are $SU(5)$ indices, and $L^T = (l_L, \nu_L)$ is a left handed lepton doublet. Y_{ij} is the Yukawa coupling constant and C is the charge-conjugation operator. Because of non-linear nature of $\Sigma_{\alpha\beta}^*$, this interaction can give rise to a mass matrix for the neutrinos as:

$$M_{ij} = Y_{ij}(\nu' + \frac{\nu^2}{4f}). \quad (2)$$

One can see from Eq.(2) that, if we would like to stabilize the Higgs mass and at the same time ensure neutrino masses consistent with experimental data[8], the coupling constant Y_{ij} must be of order 10^{-11} , which is unnaturally small. However, it has been shown[4,5] that the lepton number violating interaction only involving the complex scalar triplet Φ can give a neutrino mass matrix $M_{ij} = Y_{ij}\nu'$. Considering the current bounds on the neutrino mass[8], there should be:

$$Y_{ij}\nu' \sim 10^{-10} GeV. \quad (3)$$

Thus, the coupling constant Y_{ij} can naturally be of order one or at least not be unnaturally small provided the VEV ν' of the triplet scalar Φ is restricted to be extremely small.

In this scenario, the triplet scalar Φ has the LFV couplings to the left handed lepton pairs, which can be written as[5]:

$$\mathcal{L}_{LFV} = Y_{ij}[l_{Li}^T C^{-1} l_{Lj} \Phi^{++} + \frac{1}{\sqrt{2}}(\nu_{Li}^T C^{-1} l_{Lj} + l_{Li}^T C^{-1} \nu_{Lj}) \Phi^+ + \nu_{Li}^T C^{-1} \nu_{Lj} \Phi^0] + h.c. \quad (4)$$

Considering these LFV couplings, Ref.[5] has investigated the decays of the scalars $\Phi^{\pm\pm}$ and Φ^{\pm} , and found that the most striking signature comes from the doubly charged scalars $\Phi^{\pm\pm}$. The constraints on the coupling constant Y_{ij} and the triplet scalar mass parameter M_{Φ} coming from the muon anomalous magnetic moment a_{μ} and the LFV process $\mu^- \rightarrow e^+ e^- e^-$ are studied in Ref.[7]. In the next section, we will calculate the contributions of the charged scalars $\Phi^{\pm\pm}$ and Φ^{\pm} to the LFV processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_k$.

III. The charged scalars and the LFV processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_k$

Decay Process	Current limit	Bound(GeV^{-4})
$\mu \rightarrow e \gamma$	1.2×10^{-11} [10]	—
$\tau \rightarrow e \gamma$	1.1×10^{-7} [12]	—
$\tau \rightarrow \mu \gamma$	6.8×10^{-8} [13]	—
$\mu \rightarrow 3e$	1.0×10^{-12} [11]	$ Y_{\mu e} Y_{ee}^* ^2 / M_{\Phi}^4 \leq 2.2 \times 10^{-19}$
$\tau \rightarrow 3e$	2.0×10^{-7} [14]	$ Y_{\tau e} Y_{ee}^* ^2 / M_{\Phi}^4 \leq 2.4 \times 10^{-13}$
$\tau \rightarrow 2\mu e$	3.3×10^{-7} [14]	$ Y_{\tau e} Y_{\mu\mu}^* ^2 / M_{\Phi}^4 \leq 8.1 \times 10^{-13}$
$\tau \rightarrow 2e\mu$	2.7×10^{-7} [14]	$ Y_{\tau\mu} Y_{ee}^* ^2 / M_{\Phi}^4 \leq 6.6 \times 10^{-13}$
$\tau \rightarrow 3\mu$	1.9×10^{-7} [14]	$ Y_{\tau\mu} Y_{\mu\mu}^* ^2 / M_{\Phi}^4 \leq 2.3 \times 10^{-13}$

Table 1: The current experimental upper limits on the branching ratios of some LFV processes and the corresponding upper constraints on the free parameters.

The observation of neutrino oscillations[1] implies that the individual lepton numbers $L_{e,\mu,\tau}$ are violated, suggesting the appearance of the LFV processes, such as $l_i \rightarrow l_j \gamma$ and

$l_i \rightarrow l_j l_k l_k$. The branching ratios of these LFV processes are extremely small in the SM with right handed neutrinos. For example, Ref.[9] has shown $Br(\mu \rightarrow e\gamma) < 10^{-47}$. Such small branching ratio is unobservable.

The present experimental upper limits on the branching ratios $Br(\mu \rightarrow e\gamma)$ [10], $Br(\mu \rightarrow 3e)$ [11], $Br(\tau \rightarrow e\gamma)$ [12], $Br(\tau \rightarrow \mu\gamma)$ [13], and $Br(\tau \rightarrow l_i l_k l_k)$ [14] are given in Table 1. Future experiments with increased sensitivity can reduce these current limits by a few orders of magnitude(see, e.g.[15]). In this section, we will use these data to give the constraints on the free parameters Y_{ij} and M_Φ .

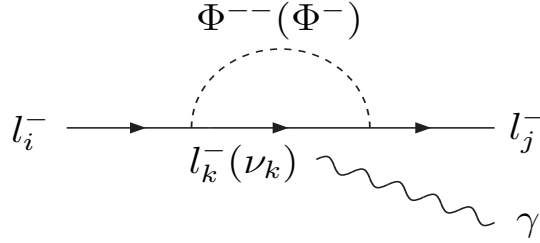


Figure 1: Feynman diagrams contributing to the radiative decay $l_i^- \rightarrow l_j^- \gamma$ due to the charged scalars $\Phi^{--}(\Phi^-)$.

The LFV couplings of the charged scalars Φ^{--} and Φ^- given in Eq.(4) can lead to the LFV radiative decays $l_i^- \rightarrow l_j^- \gamma$ at the one loop level mediated by the exchange of the charged scalars Φ^{--} and Φ^- , as shown in Fig.1. For the doubly charged scalar Φ^{--} , the photon can be attached either to the internal lepton line or to the scalar line. For the charged scalar Φ^- , the photon can be only attached to the scalar line[16].

Using Eq.(4), the expression of the branching ratio $Br(l_i^- \rightarrow l_j^- \gamma)$ can be written as at leading order:

$$Br(l_i^- \rightarrow l_j^- \gamma) = \frac{\alpha_e}{96\pi G_F^2} \sum_{k=\tau,\mu,e} (Y_{ik} Y_{kj}^*)^2 \left[\frac{3\delta_{ki(j)} + 1}{M_{\Phi^{--}}^2} + \frac{1}{M_{\Phi^-}^2} \right]^2 Br(l_i \rightarrow e \nu_e \bar{\nu}_i). \quad (5)$$

Where α_e is the fine structure constant and G_F is the Fermi constant. The factor $3\delta_{ki(j)}$ means that, when the internal lepton is the same as one of the leptons l_i and l_j , the contributions of Φ^{--} to $Br(l_i^- \rightarrow l_j^- \gamma)$ is four times those for $k \neq i$ and j . $M_{\Phi^{--}}$ and

M_{Φ^-} are the masses of the scalars Φ^{--} and Φ^- , respectively. In the LH model, the scalar mass is generated through the Coleman-Weinberg mechanism and the scalars Φ^{--} , Φ^- and Φ^0 degenerate at the lowest order[5]. Thus, we can assume $M_{\Phi^{--}} = M_{\Phi^-}$ and write the branching ratio as:

$$Br(l_i^- \rightarrow l_j^- \gamma) = \frac{\alpha_e}{96\pi G_F^2 M_\Phi^4} \sum_{k=\tau,\mu,e} (Y_{ik} Y_{kj}^*)^2 [3\delta_{ki(j)} + 2]^2 Br(l_i \rightarrow e \nu_e \bar{\nu}_i). \quad (6)$$

In particular, for the decay process $\mu^- \rightarrow e^- \gamma$, we obtain the following expression for the branching ratio $Br(\mu^- \rightarrow e^- \gamma)$:

$$Br(\mu^- \rightarrow e^- \gamma) = \frac{\alpha_e}{96\pi G_F^2 M_\Phi^4} [25(Y_{\mu e} Y_{ee}^*)^2 + 25(Y_{\mu\mu} Y_{\mu e}^*)^2 + 4(Y_{\mu\tau} Y_{\tau e}^*)^2]. \quad (7)$$

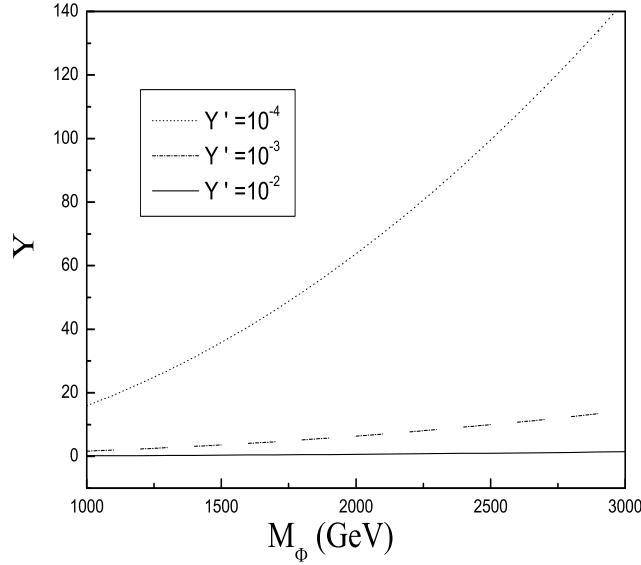


Figure 2: The FD coupling constant Y as a function of the scalar mass M_Φ for different values of the FX coupling constant Y' .

From above equations, we can see that the LFV process $l_i \rightarrow l_j \gamma$ can not be able to constrain Y_{ij} independently. However, if we assume $Y_{ik} = Y$ for $i = k$ (Y is the flavor-diagonal (FD) coupling constant) and $Y_{ik} = Y'$ for $i \neq k$ (Y' is the flavor-mixing (FX) coupling constant), then we can obtain the constraints on the combination of the free

parameters Y , Y' and M_Φ . Observably, the most stringent constraint should come from the current experimental upper limits on the branching ratio $Br(\mu \rightarrow e\gamma)$. Thus, in Fig.2, we have shown the FD coupling constant Y as a function of the mass parameter M_Φ for $Y' = 1 \times 10^{-2}$, 1×10^{-3} and 1×10^{-4} . From Fig.2, one can see the upper limit on Y strongly depend on the values of M_Φ and Y' . For $M_\Phi \leq 2000 GeV$ and $Y' \geq 1 \times 10^{-4}$, there must be $Y \leq 64$.

In the LH model, the LFV processes $l_i \rightarrow l_j l_k l_k$ can be generated at tree level through the exchange of doubly charged scalar $\Phi^{\pm\pm}$, as depicted in Fig.3.

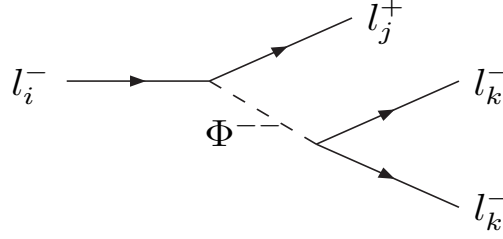


Figure 3: Tree level Feynman diagram for the LFV processes $l_i^- \rightarrow l_j^+ l_k^- l_k^-$ mediated by the doubly charged scalar Φ^{--} .

The expressions of the branching ratios for the processes $l_i^- \rightarrow l_j^+ l_k^- l_k^-$ are given by [16,17]

$$Br(\mu^- \rightarrow e^+ e^- e^-) = \frac{|Y_{\mu e} Y_{ee}^*|^2}{16G_F^2 M_\Phi^4}, \quad (8)$$

$$Br(\tau^- \rightarrow e^+ e^- e^-) = \frac{|Y_{\tau e} Y_{ee}^*|^2}{16G_F^2 M_\Phi^4} Br(\tau \rightarrow e \nu_e \bar{\nu}_\tau), \quad (9)$$

$$Br(\tau^- \rightarrow \mu^+ e^- e^-) = \frac{|Y_{\tau \mu} Y_{ee}^*|^2}{32G_F^2 M_\Phi^4} Br(\tau \rightarrow e \nu_e \bar{\nu}_\tau), \quad (10)$$

$$Br(\tau^- \rightarrow e^+ \mu^- \mu^-) = \frac{|Y_{\tau e} Y_{\mu \mu}^*|^2}{32G_F^2 M_\Phi^4} Br(\tau \rightarrow e \nu_e \bar{\nu}_\tau), \quad (11)$$

$$Br(\tau^- \rightarrow \mu^+ \mu^- \mu^-) = \frac{|Y_{\tau \mu} Y_{\mu \mu}^*|^2}{16G_F^2 M_\Phi^4} Br(\tau \rightarrow e \nu_e \bar{\nu}_\tau). \quad (12)$$

Certainly, up to one loop, the LFV processes $l_i \rightarrow l_j l_k l_k$ get additional contributions from the processes $l_i \rightarrow l_j \gamma^* \rightarrow l_j l_k l_k$. Thus, the charged scalars $\Phi^{\pm\pm}$ and Φ^\pm have contributions to the LFV processes $l_i \rightarrow l_j l_k l_k$ at one loop. However, compared with the tree level contributions, they are very small, which can be safely neglected.

The LFV processes $l_i \rightarrow l_j l_k l_k$ also can not give the constraints on the coupling constants Y_{ij} independently, but would be able to constrain the combination $|Y_{ij} Y_{kk}^\dagger|^2 / M_\Phi^4$. Our numerical results are given in Table 1.

In the following section, we will take into account these constraints coming from the LFV processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_k$, estimate the contributions of the doubly charged scalars $\Phi^{\pm\pm}$ to the processes $e^\pm e^\pm \rightarrow l_i^\pm l_j^\pm$ and $e^+ e^- \rightarrow l_i^\pm l_j^\pm$, and discuss the possibility of detecting the signals for the doubly charged scalars $\Phi^{\pm\pm}$ at the ILC experiments.

IV. The doubly charged scalars $\Phi^{\pm\pm}$ and the LFV processes $e^\pm e^\pm \rightarrow l_i^\pm l_j^\pm$ and $e^+ e^- \rightarrow l_i^\pm l_j^\pm$

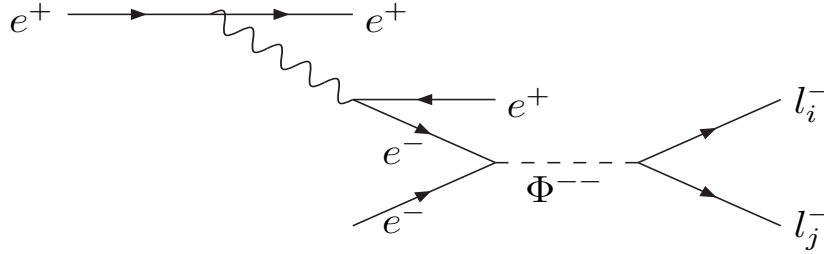


Figure 4: Main Feynman diagram for the processes $e^+ e^- \rightarrow l_i^- l_j^-$ predicted by Φ^{--} .

In general, the doubly charged scalars can not couple to quarks and their couplings to leptons break the lepton number by two units, leading to a distinct signature, namely a pair of same sign leptons. The discovery of a doubly charged scalar would have important implications for our understanding of the Higgs sector and more importantly, for what lies beyond the SM . This fact has made one give more elaborate theoretical calculations in the framework of some specific models beyond the SM and see whether the signatures of this kind of new particles can be detected in the future high energy experiments. For example, the production and decay of the doubly charged scalars and their possible signals at the ILC have been extensively studied in Refs.[18,19]. In this section, we will consider the contributions of the doubly charged scalars $\Phi^{\pm\pm}$ predicted by the LH model to the

processes $e^\pm e^\pm \rightarrow l_i^\pm l_j^\pm$ and $e^+ e^- \rightarrow l_i^\pm l_j^\pm$ (l_i or $l_j \neq e$). The processes $e^\pm e^\pm \rightarrow l_i^\pm l_j^\pm$ can be seen as the subprocesses of the processes $e^+ e^- \rightarrow l_i^\pm l_j^\pm$. For example, the doubly charged scalar Φ^{--} generates contributes to the process $e^+ e^- \rightarrow l_i^- l_j^-$ through the subprocess $e^- e^- \rightarrow l_i^- l_j^-$, as shown in Fig.4.

Using Eq.(4), the expression of the cross section for the subprocess $e^- e^- \rightarrow l_i^- l_j^-$ can be easily written as:

$$\hat{\sigma}(\hat{s}) = \frac{Y_{ee}^2 Y_{ij}^2}{8\pi} \frac{\hat{s}}{(\hat{s} - M_\Phi^2)^2 + M_\Phi^2 \Gamma_\Phi^2}. \quad (13)$$

Where $\sqrt{\hat{s}}$ is the center-of-mass (*C.M.*) energy of the subprocess $e^- e^- \rightarrow l_i^- l_j^-$. Γ_Φ is the total decay width of the doubly charged scalar Φ^{--} , which has been given by Ref.[5] in the case of the triplet scalars ($\Phi^{\pm\pm}$, Φ^\pm , and Φ^0) degenerating at lowest order with a common mass M_Φ :

$$\begin{aligned} \Gamma_\Phi &= \sum_{ij} \Gamma(\Phi^{--} \rightarrow l_i^- l_j^-) + \Gamma(\Phi^{--} \rightarrow W_L^- W_L^-) + \Gamma(\Phi^{--} \rightarrow W_T^- W_T^-) \\ &\approx \frac{M_\Phi}{8\pi} [3Y^2 + 6Y'^2] + \frac{\nu'^2 M_\Phi^3}{2\pi\nu'^4} + \frac{g^4 \nu'^2}{4\pi M_\Phi}. \end{aligned} \quad (14)$$

Where $Y = Y_{ij}$ ($i = j$) is the *FD* coupling constant, $Y' = Y_{ij}$ ($i \neq j$) is the *FX* coupling constant. In above equation, the final-state masses have been neglected compared to the mass parameter M_Φ . It has been shown that, for $\nu' < 1 \times 10^{-5}$, the main decay modes of Φ^{--} are $l_i^- l_j^-$. Furthermore, the *FX* coupling constant Y' are subject to very stringent bounds from the *LFV* process $\mu \rightarrow eee$. In this case, the decay width Γ_Φ can be approximately written as:

$$\Gamma_\Phi \approx \frac{3M_\Phi Y^2}{8\pi}. \quad (15)$$

Considering the current bounds on the neutrino mass[8], there should be:

$$Y_{ij} \nu' \sim 10^{-10} GeV, \quad (16)$$

so $\nu' < 1 \times 10^{-5}$ leads to $Y_{ij} > 1 \times 10^{-5}$, which does not conflict with the most stringent constraint from the *LFV* process $\mu \rightarrow eee$. Thus, in our numerical calculation, we will take Eq.(15) as the total decay width of Φ^{--} .

Using the equivalent particle approximation method[20], the effective cross section for the process $e^+e^- \rightarrow l_i^- l_j^-$ can be approximately written as[19]:

$$\sigma(E_{e^+}, s) = \int_{x_{min}}^1 dx F_{e^+}^{e^-}(x, E_{e^+}) \hat{\sigma}(\hat{s}). \quad (17)$$

Where $\hat{s} = xs$ and $x_{min} = (m_{l_i} + m_{l_j})^2/s$. $F_{e^+}^{e^-}(x, E_{e^+})$ is the equivalent electron distribution function of the initial positron, which gives the probability that an electron with energy $E_{e^-} = xE_{e^+}$ is emitted from a positron beam with energy E_{e^+} . The relevant expression can be written as[21]:

$$F_{e^+}^{e^-}(x, E_{e^+}) = \frac{\alpha_e^2}{8\pi^2 x} [\ln(\frac{E_{e^+}}{m_e})^2 - 1]^2 [\frac{4}{3} + x - x^2 - \frac{4}{3}x^3 + 2x(1+x)\ln x]. \quad (18)$$

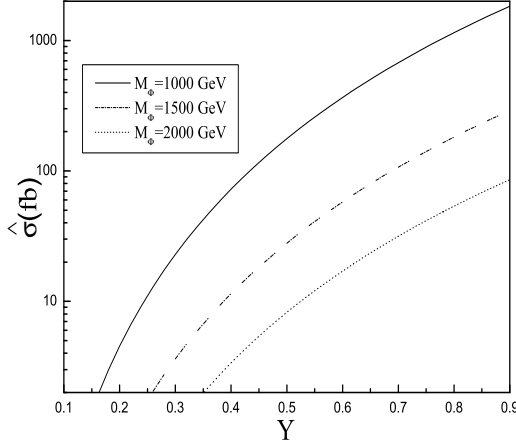


Figure 5: The cross section $\hat{\sigma}(\hat{s})$ as a function of Y for three values of the mass M_Φ .

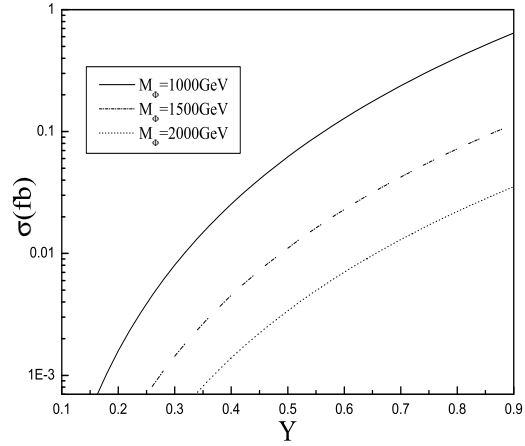


Figure 6: Same as Fig.5 but for $\sigma(s)$.

In Fig.5 and Fig.6, we plot the production cross sections $\hat{\sigma}(\hat{s})$ and $\sigma(s)$ for the processes $e^-e^- \rightarrow \mu^-\mu^-$ and $e^+e^- \rightarrow \mu^-\mu^-$ as function of the FD coupling constant Y , respectively. In these figures, we have assumed $0.15 \leq Y \leq 0.9$ and taken $\sqrt{s} = 500\text{GeV}$ and $M_\Phi = 1.0\text{TeV}, 1.5\text{TeV}, 2.0\text{TeV}$. From Fig.5 and Fig.6 one can see that the values of $\hat{\sigma}(\hat{s})$ and $\sigma(s)$ are strongly depend on the value of the FD coupling constant $Y(Y_{ee})$. For $Y \geq 0.7$ and $M_\Phi \leq 1.5\text{TeV}$, the values of the subprocess cross section $\hat{\sigma}(\hat{s})$ and the effective cross section $\sigma(s)$ are larger than $1.1 \times 10^2 \text{ fb}$ and $4.3 \times 10^{-2} \text{ fb}$, respectively.

The signal of the doubly charged scalar Φ^{--} given by the process $e^+e^- \rightarrow \mu^-\mu^-$ is so distinctive and is the SM background free, discovery would be signalled by even few events. In Fig.7, we plot the discovery region in the $Y - M_\Phi$ plane at 95% confidence level ($C.L.$) for seeing 5 $\mu^-\mu^-$ events, in which we have assumed the future ILC with the $C.M.$ energy $\sqrt{s} = 500GeV$ and the yearly integrated luminosity of $\mathcal{L} = 500fb^{-1}$ [22]. From this figure, one can see that, in wide range of the parameter space, the signals of Φ^{--} should be detected in the future ILC experiments.

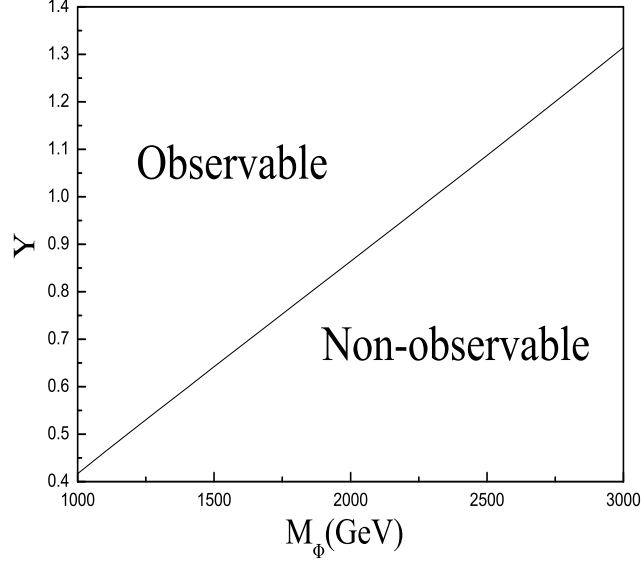


Figure 7: Discovery region in the $Y - M_\Phi$ plane at 95% $C.L.$ for seeing 5 $\mu^-\mu^-$ events.

The doubly charged scalar Φ^{--} can also has contributions to the LFV processes $e^+e^- \rightarrow \tau^-\mu^-, \tau^-e^-,$ and μ^-e^- . However, the experimental upper limits on the LFV processes $\tau \rightarrow \mu ee, \tau \rightarrow eee,$ and $\mu \rightarrow eee$ can give severe constraints on the combination $|Y_{ij}Y_{kk}^\dagger|^2/M_\Phi^4$, which makes the production cross sections of these processes very small. For example, even if we take $Y = 1$ and $M_\Phi \leq 2TeV$, the production cross sections $\sigma(\tau\mu), \sigma(\tau e),$ and $\sigma(\mu e)$ are smaller than $6.9 \times 10^{-3}fb, 2.1 \times 10^{-3}fb,$ and $1.9 \times 10^{-9}fb,$ respectively. Thus, it is very difficult to detect the signals of Φ^{--} via the processes $e^+e^- \rightarrow l_i^-l_j^- (i \neq j)$ in the future ILC experiments.

Certainly, the doubly charged scalar Φ^{++} has contributions to the processes $e^+e^+ \rightarrow$

$l_i^+ l_j^+$ and $e^+ e^- \rightarrow l_i^+ l_j^+$. Similar with above calculation, we can give the values of the production cross sections for these processes. We find that the cross section $\sigma(l_i^+ l_j^+)$ is equal to the cross section $\sigma(l_i^- l_j^-)$. Thus, the conclusions for the doubly charged scalar Φ^{--} are also apply to the doubly charged scalar Φ^{++} .

V. Conclusions

To solve the so-called hierarchy or fine tuning problem of the *SM*, the little Higgs theory was proposed as a kind of models to *EW**SB* accomplished by a naturally light Higgs boson. The *LH* model is one of the simplest and phenomenologically viable models. In the *LH* model, neutrino masses and mixings can be generated by coupling the scalar triplet Φ to the leptons in a $\Delta L = 2$ interaction, whose magnitude is proportional to the triplet $VEV\nu'$ multiplied by the Yukawa coupling constant Y_{ij} without invoking a right handed neutrino. This scenario predicts the existence of doubly charged scalars $\Phi^{\pm\pm}$. For smaller values of ν' i.e. $\nu' \leq 1 \times 10^{-5}$, the doubly charged scalars $\Phi^{\pm\pm}$ have large flavor changing coupling to leptons, which can generate significantly contributions to some *LFV* processes and give characteristic signatures in the future high energy experiments.

In this paper, we first consider the *LFV* processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_k$ in the context of the *LH* model. For the *LFV* process $l_i \rightarrow l_j \gamma$, it involves all of the FX coupling constants $Y_{ij}(i \neq j)$, we can not give the simple constraints about the free parameters Y_{ij} and M_Φ . Thus, for the fixed values of the FX coupling constant $Y' = Y_{ij}(i \neq j)$, we take into account the current experimental upper limit of the *LFV* $\mu \rightarrow e \gamma$ and plot the FD coupling constant $Y = Y_{ij}(i = j)$ as a function of the mass parameter M_Φ . Our numerical results show that the upper limit on Y is strongly depend on the free parameters M_Φ and Y' .

Using the present experimental upper limits on the branching ratios $Br(l_i \rightarrow l_j l_k l_k)$, we obtain the constraints on the combination $|Y_{ij} Y_{kk}^*|^2 / M_\Phi^4$. We find that the most stringent constraint comes from the *LFV* process $\mu \rightarrow e e e$. In all of the parameter space, there must be $|Y_{\mu e} Y_{ee}^*|^2 / M_\Phi^4 \leq 2.2 \times 10^{-19} GeV^{-4}$.

The characteristic signals of the processes $e^+e^- \rightarrow l_i^\pm l_j^\pm$ is same-sign dileptons or two same-sign different flavor leptons, which is the SM background free and offers excellent potential for doubly charged scalar discovery. To see whether the doubly charged scalar Φ^{--} can be detected in the future ILC experiments, we discuss the contributions of Φ^{--} to the processes $e^-e^- \rightarrow l_i^- l_j^-$ and $e^+e^- \rightarrow l_i^- l_j^-$. We find that the triplet scalar Φ^{--} can give significantly contributions to the processes $e^+e^- \rightarrow l_i^- l_i^-$. In wide range of the parameter space of the LH model, the possible signals of Φ^{--} might be observed in the future ILC experiments. However, the production cross sections of the LFV processes $e^+e^- \rightarrow l_i^- l_j^- (i \neq j)$ mediated by Φ^{--} are very small. The contributions of the triplet scalar Φ^{++} to the processes $e^+e^- \rightarrow l_i^+ l_j^+$ are equal to those of Φ^{--} for the processes $e^+e^- \rightarrow l_i^- l_j^-$, Thus, our conclusions are also apply to the doubly charged scalar Φ^{++} .

Some popular models beyond the SM predict the existence of doubly charged scalars, which generally have the lepton number and lepton flavor changing couplings to leptons and might produce distinct experimental signatures in the current or future high energy experiments. Their observation would signal physics outside the current paradigm and further test the new physics models. Search for this kind of new particles has been one of the important goals of the high energy experiments[23]. Thus, the possibly signals of the doubly charged scalars $\Phi^{\pm\pm}$ predicted by the little Higgs models should be more studied in the future.

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